LAST NAME: Apr 1 / 2013

FIRST NAME: Solution

**Problem 1** Let $L_1$ be the language accepted by the finite automaton given on Figure 1.

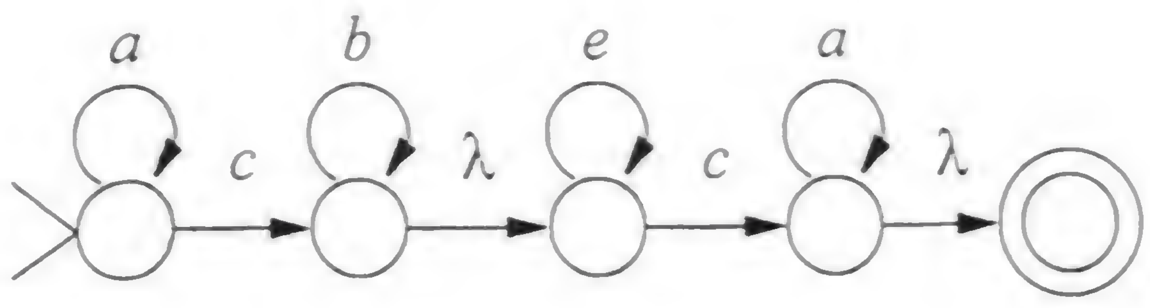



Figure 1: Language $L_1$

Let $L_2$ be the language generated by the context-free grammar $G_2 = (V, \Sigma, P, S_2)$, where $\Sigma = \{a, b, c, e\}$, $V = \{S_2\}$, and the production set $P$ is:

$$S_2 \rightarrow aS_2a \mid bS_2b \mid cS_2c \mid \lambda$$

(a) Write 5 distinct strings that belong to $L_1$ but do not belong to $L_2$ (belong to $L_1 \setminus L_2$). If such strings do not exist, state it and explain why.

**Answer:**

acc, acbc, cca, accaa

(b) Write 5 distinct strings that belong to $L_2$ but do not belong to $L_1$ (belong to $L_2 \setminus L_1$). If such strings do not exist, state it and explain why.

**Answer:**

$\lambda$, aa, bb, abba, baab

(c) Write 5 distinct strings that belong to $L_1$ and $L_2$ (belong to $L_1 \cap L_2$). If such strings do not exist, state it and explain why.

**Answer:**

cc, acca, aaccaa, cbbc, acbbca

(d) Write 5 distinct strings over alphabet $\{a, b, c, d\}$ that do not belong to $L_1$ and do not belong to $L_2$ (belong to $\overline{L_1} \cap \overline{L_2}$). If such strings do not exist, state it and explain why.

**Answer:**

ccc, c, bcb, ba, cb

(e) Write 5 distinct strings that belong to $a^*c^*b^*e^*c^*a^*$ but do not belong to $L_1$ (belong to $a^*c^*b^*e^*c^*a^* \setminus L_1$.) If such strings do not exist, state it and explain why.

**Answer:**

$\lambda$, c, ccc, ab, a

(f) Write 5 distinct strings that belong to $L_1$ but have a length equal to 3. If such strings do not exist, state it and explain why.

**Answer:**

only 4 exist:

acc, cbc, cec, cca

LAST NAME:

FIRST NAME: Solution

**Problem 3** (a) Calculate the image of the sequence $\langle 3,0,1,1\rangle$ under Gödel numbering and show your work. If this image does not exist, state it and explain why.

**Answer:**

$g(\langle 3,0,1,1\rangle) =$

$2^{3+1} \cdot 3^{0+1} \cdot 5^{1+1} \cdot 7^{1+1} =$

$16 \cdot 3 \cdot 25 \cdot 49 =$

$4 \cdot 100 \cdot 3 \cdot 49 =$

$400 \cdot 147 =$

$\boxed{58800}$

(b) Calculate the pre-image (original) of the number 5880 under Gödel numbering and show your work. If this pre-image does not exist, state it and explain why.

**Answer:**

$5880 = 2 \cdot 2940$

$= 2 \cdot 2 \cdot 1470$

$= 2 \cdot 2 \cdot 2 \cdot 735$

$= 2 \cdot 2 \cdot 2 \cdot 3 \cdot 245$

$= 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5 \cdot 49$

$= 2^3 \cdot 3 \cdot 5 \cdot 7^2$

$g^{-1}(5880) =$

$\boxed{\langle 2,0,0,1\rangle}$

In each of the cases below, state the cardinality of the given set. If this cardinality is finite, state the exact number; if it is infinite, specify whether it is countable or uncountable.

(c) set whose regular expression over $\Sigma = \{a, b\}$ is:

$$\emptyset \cup a$$

**Answer:** 1

(d) set whose regular expression over $\Sigma = \{a, b\}$ is:

$$\emptyset^* \cup a^*$$

**Answer:** infinite and countable

(e) set whose regular expression over $\Sigma = \{a, b\}$ is:

$$\emptyset^* \cup a$$

**Answer:** 2

(f) set whose regular expression over $\Sigma = \{a, b\}$ is:

$$\emptyset^* a$$

**Answer:** 1

(g) set whose regular expression over $\Sigma = \{a, b\}$ is:

$$\emptyset a$$

**Answer:** 0

(h) set whose regular expression over $\Sigma = \{a, b\}$ is:

$$\emptyset^* \cup \lambda$$

**Answer:** 1

(i) class of languages over $\Sigma = \{a, b\}$ that are regular;
**Answer:** infinite and countable

LAST NAME:

FIRST NAME: Solution

**Problem 5** Let $L$ be the set of all strings over the alphabet $\{a, b, c\}$ which satisfy all of the following conditions:

1. does not begin with $c$;
2. does not end end with $a$;
3. has an odd length.

(a) Write 5 distinct strings that belong to $L$. If such strings do not exist, state it and explain why.

b, bbb, abc, bcb, aab

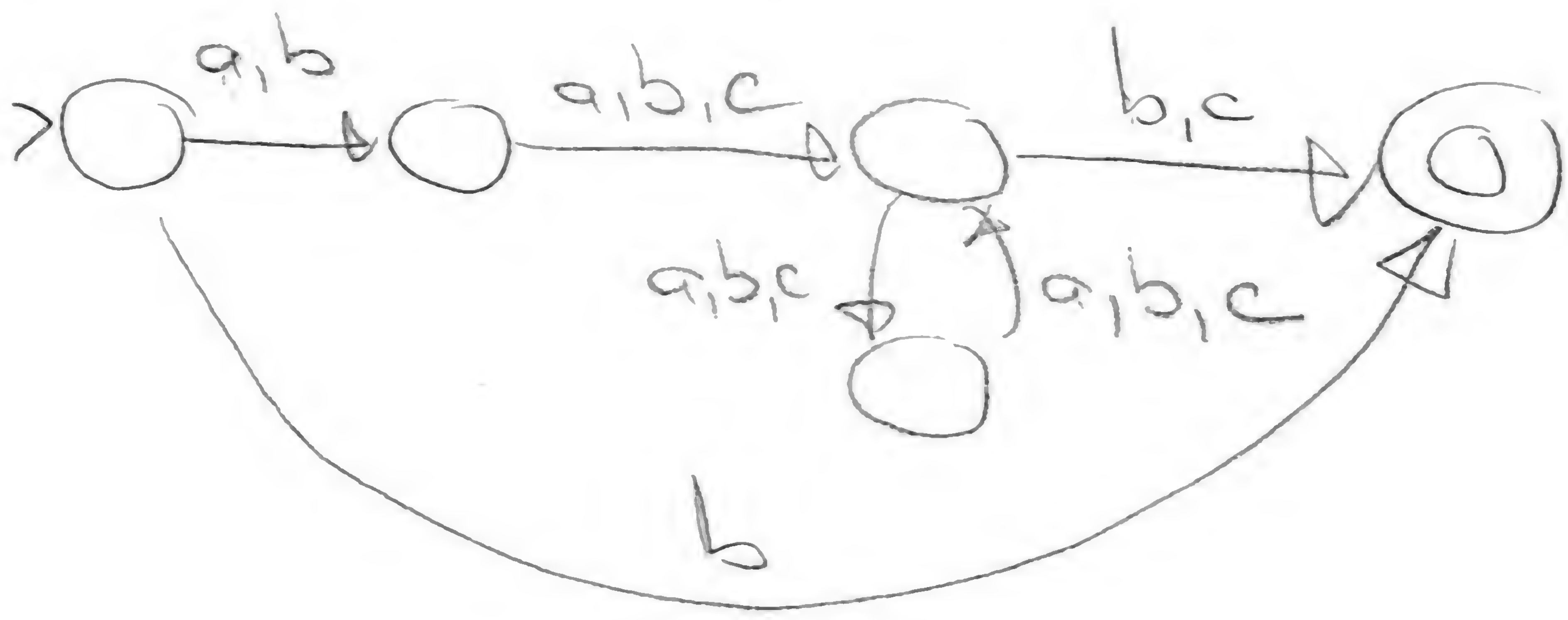
(b) Write a regular expression that represents the language $L$. If such a regular expression does not exist, state it and explain why.

**Answer:**

$b \cup (a \cup b)(a \cup b \cup c)((a \cup b \cup c)(a \cup b \cup c))^*(b \cup c)$

(c) Draw a state-transition graph of a finite automaton that accepts the language $L$. If such an automaton does not exist, state it and explain why.

**Answer:**




(d) Write a complete formal definition of a context-free grammar that generates the language $L$. If such a grammar does not exist, state it and explain why.

**Answer:**

$G = (V, \Sigma, P, S)$, $\Sigma = \{a, b, c\}$

$V = \{S, A, B, Z, E\}$

$P$: $S \to b \mid AZEB$

$A \to a \mid b$

$B \to b \mid c$

$Z \to a \mid b \mid c$

$E \to \lambda \mid EE \mid ZZ$

**Problem 7** Let $L$ be the set of all strings over the alphabet $\{a, b, c\}$ which satisfy all of the following properties.

1. if the string does not contain any $b$'s then its length is even and it contains exactly one $a$;
2. if the string contains a positive even number of $b$'s, then its length is odd and it contains exactly one $c$;
3. if the string contains an odd number of $b$'s, then it is a palindrome.

Write a complete formal definition of a context-free grammar that generates the language $L$. If such a grammar does not exist, state it and explain why.

LAST NAME:

FIRST NAME: Solution

N : no c
Y : seen c

first index : count a
second index : count b

**Answer:**

$G = (V, \Sigma, P, S)$, $\Sigma = \{a, b, c\}$

$V = \{S, S_1, S_2, S_3, E, N_{00}, N_{01}, N_{02}, Y_{00}, Y_{01}, Y_{02}, N_{10}, N_{11}, N_{12}, Y_{10}, Y_{11}, Y_{12}\}$

P:

$S \to S_1 \mid S_2 \mid S_3$

$S_1 \to EacE \mid EcaE$

$E \to ccE \mid \lambda$

$S_3 \to aS_3a \mid bS_3b \mid cS_3c \mid b$

$S_2 \to N_{00}$, $Y_{02} \to \lambda$

$N_{00} \to aN_{10} \mid bN_{01} \mid cY_{00}$ $\quad$ $N_{10} \to aN_{00} \mid bN_{11} \mid cY_{10}$

$N_{01} \to aN_{11} \mid bN_{02} \mid cY_{01}$ $\quad$ $N_{11} \to aN_{01} \mid bN_{12} \mid cY_{11}$

$N_{02} \to aN_{12} \mid bN_{01} \mid cY_{02}$ $\quad$ $N_{12} \to aN_{02} \mid bN_{11} \mid cY_{12}$

$Y_{00} \to aY_{10} \mid bY_{01}$ $\quad$ $Y_{10} \to aY_{00} \mid bY_{11}$

$Y_{01} \to aY_{11} \mid bY_{02}$ $\quad$ $Y_{11} \to aY_{01} \mid bY_{12}$

$Y_{02} \to aY_{12} \mid bY_{01}$ $\quad$ $Y_{12} \to aY_{02} \mid bY_{11}$